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Measurement Uncertainty in Planetary Boundary Layer Height via Model-Based Monte Carlo Simulation

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ABSTRACT:

The planetary boundary layer height (PBLH) is a key parameter for understanding atmospheric exchange processes. While various methods exist to estimate PBLH from radiosonde profiles, the uncertainty in these estimates is often neglected. This study introduces a framework to propagate measurement uncertainties from RS41-GDP.1 radiosonde data into PBLH estimates. A Model-Based Monte Carlo simulation generates synthetic profiles by perturbing the vertical profile according to measurement uncertainties. Each profile yields a PBLH estimate, and the ensemble provides a mean and standard deviation representing expected value and uncertainty. Preliminary results using a simple modelling assumption of independent noise reveal limitations: unrealistic profile smoothness, compromised especially gradient-based methods, and inconsistencies between Monte Carlo and original estimates. These findings highlight the need for a more robust approach. Future work will incorporate geostatistical modeling to account for spatial autocorrelation and cross-variable dependencies, enabling more realistic synthetic profiles and stable uncertainty quantification. This refined methodology aims to enhance the reliability of PBLH estimates and support their application in atmospheric research and modeling.

1. MOTIVATION & RESEARCH QUESTION:

The **planetary boundary layer** (PBL), being the lowest portion of the troposphere and directly influenced by interactions with the Earth's surface, plays a crucial role in various applications and phenomena of interest. A key parameter is the **PBLH**, which defines the altitude at which exchanges occur between the PBL and the free atmosphere.

Several **methods** have been proposed in the literature to estimate PBLH using atmospheric data from different sources. Among these sources, vertical profiles obtained from **radiosondes** have proven particularly useful for investigating the atmospheric vertical structure.

Despite its relevance, the **uncertainty** associated with PBLH estimates has not been fully addressed. A rigorous treatment of any physical quantity requires a quantitative expression of the uncertainty involved.

The **research question** can be expressed as follows. How can the measurement uncertainties of radiosonde data be propagated to the PBLH estimates of a given method applied to a fixed vertical profile?

2. METHODOLOGY:

The **vertical profile** of the atmosphere can be represented using a **continuous function**, which maps thermodynamic and dynamic variables as functions of time and altitude, formally:

$$\boldsymbol{x}_{(t,z)} = \left[z_{(t,z)}, T_{(t,z)}, p_{(t,z)}, RH_{(t,z)}, wspeed_{(t,z)}, wdir_{(t,z)}\right]$$

where t and z denote a timestamp and altitude, while elements of the vector are respectively: altitude, temperature, pressure, relative humidity, wind speed, and direction.

A vector of **intermediate variables** can be determined from a vertical profile, by a given function f, denoted by:

$$\boldsymbol{y}_{(t,z)} = \left[\Gamma_{\theta_{(t,z)}}, \Gamma_{RH_{(t,z)}}, Ri_{(t,z)}\right] = f(\boldsymbol{x}_{(t,z)})$$

with elements: vertical gradient of virtual potential temperature, vertical gradient of relative humidity, and Richardson number.

PBLH methods are then formalized as follows:

- **Parcel method** (PM) $\quad z^*_{pm} = inf\{z: \theta_{(t,z)} - \theta_s \geq 0\}$
- **Maximum gradient of θ** (θ) $\quad z^*_{\theta} = argmax_z\left(\Gamma_{\theta_{(t,z)}}, z < z_{max}\right)$
- **Minimum gradient of RH** (RH) $\quad z^*_{RH} = argmin_z\left(\Gamma_{RH_{(t,z)}}, z < z_{max}\right)$
- **Richardson number method** (Ri) $\quad z^*_{Ri} = inf\{z: Ri_{(t,z)} > Ri_c\}$

where z_{max} is the altitude upper bound for PBLH, and $Ri_c = 0.25$ is the Richardson critical value.

Each radiosonde measurement can be modeled by a **measurement equation**:

$$\hat{\boldsymbol{x}}_t = \boldsymbol{x}_{(t,z)} + \boldsymbol{\epsilon}_t$$

ile is **discretized** at regular time intervals, $t = 1, \ldots, N$, and a **measurement error**, $\boldsymbol{\epsilon}_t$ p-variate, is added to the true vertical profile. The **measured vertical profile** can be denoted by $X = \{\hat{\boldsymbol{x}}_t\}_{t=1}^N$ and the **error profile** by $\epsilon = \{\boldsymbol{\epsilon}_t\}_{t=1}^N$.

Common approaches to obtain an estimate of intermediate variables ($\boldsymbol{y}_{(t,z)}$) from the measured vertical profile (X) are: to compute gradient as the **finite difference ratio**, and to use **Bulk Richardson number,** that is

$$Y = \{\hat{\boldsymbol{y}}_t\}_{t=1}^N = g(X)$$

and then apply the PBLH methods on the elements of Y.

The proposed methodology is to use a **Model-Based Monte Carlo approach**:

1. Obtain a **model** for the vertical profile of the atmosphere ($\boldsymbol{x}_{(t,z)}$) and the error profile (ϵ)
2. Generate a sample of X_i with $i = 1, \ldots, M$ **synthetic profiles**. Each profile is obtained by perturbing the modeled $\boldsymbol{x}_{(t,z)}$ according to stochastic realization of ϵ.
3. Apply the chosen **PBLH method** to each of the X_i profiles independently. This yields an ensemble of M PBLH values.
4. Compute the **sample mean** and **standard deviation** of the resulting PBLH values. The mean provides the expected PBLH, while the standard deviation quantifies its uncertainty due to measurement uncertainty.

This methodology is based on the ability to accurately model the vertical profile and the measurement error.

5. OPEN-SOURCE:

Would you like to replicate, test, or extend these experiments?

I am developing an **open-source Python package** to work with GDPs.

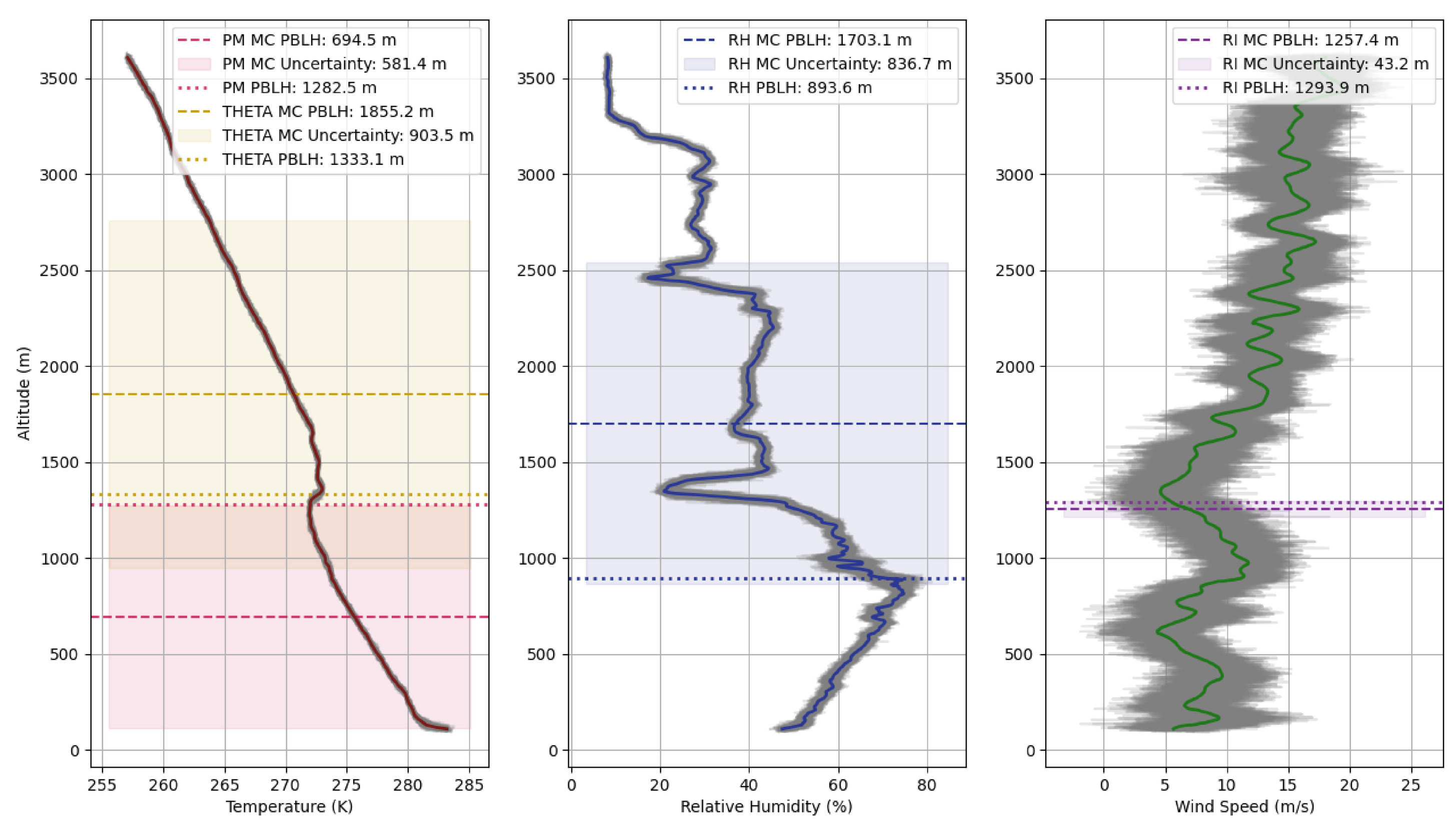
SCAN ME

3. PRELIMINARY ANALYSIS:

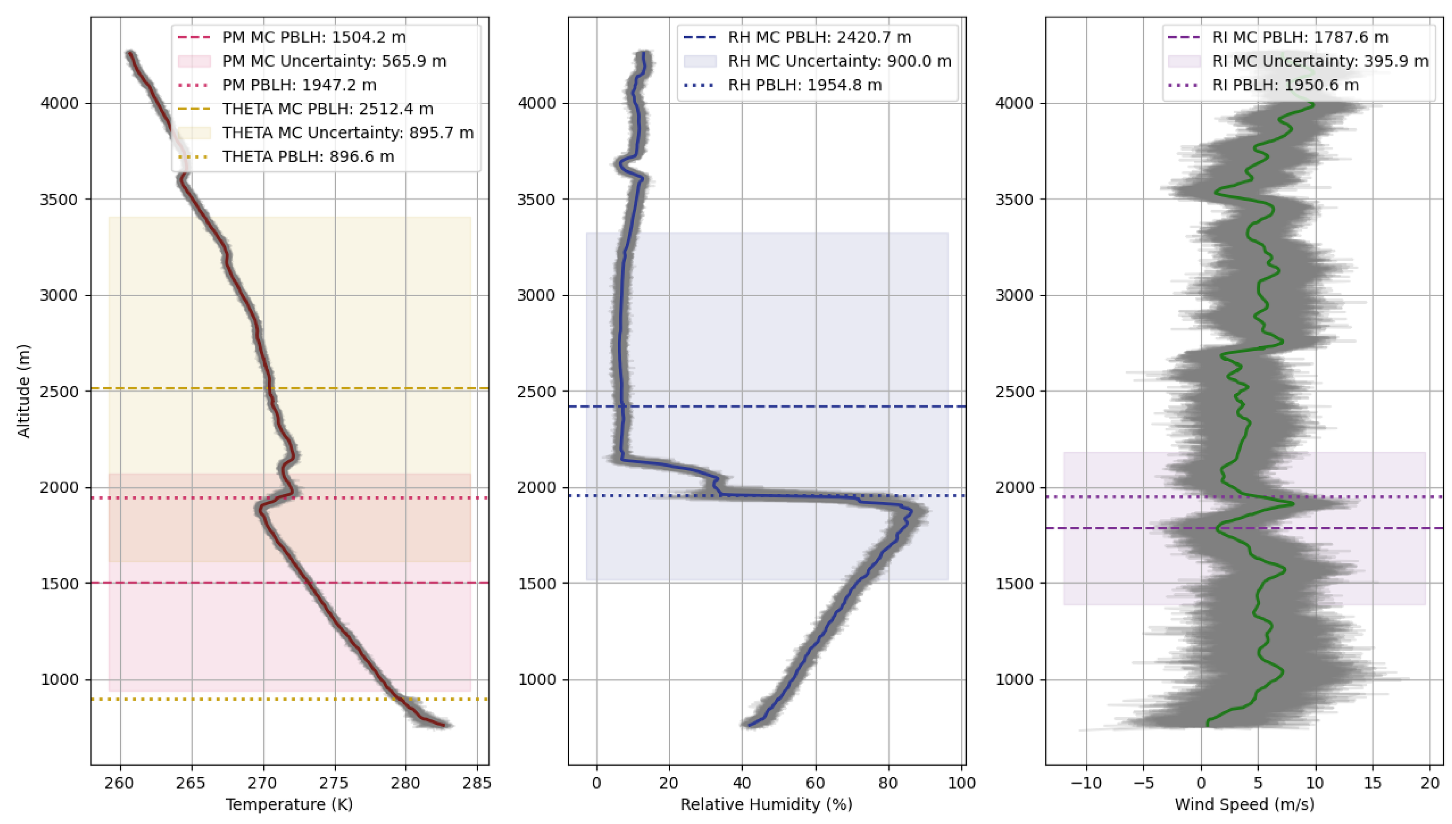
A simple **modelling approach,** is to consider: $\boldsymbol{x}_{(t,z)} = \hat{\boldsymbol{x}}_t$, and $\boldsymbol{\epsilon}_t$ to be a 6-dimensional vector of **independent Gaussian white noise** components, i.e., no autocorrelation and cross-correlation.

Each noise component is characterized by its standard deviation. The **RS41-GDP.1** radiosonde is employed because it provides vertically resolved uncertainty estimates for each measurement, reported as expanded uncertainties with a coverage factor of $k = 2$.

RS41-GDP: Lindenberg, 2017-03-03T10:58:21.278Z



RS41-GDP: Potenza, 2025-03-19T14:21:17.967Z



Figures: original measurement (colored lines), synthetic profiles (gray lines)

Take-aways:

- This modeling approach seems to build unrealistic profiles (loss of smoothness) compromising gradient methods
- PLBH methods suffer from the high resolution of these GDPs,
- MC estimate is not always in accordance with the original estimate.

4. DISCUSSION & FUTURE WORKS:

This modelling approach has several **limitations** in creating realistic synthetic profiles. Satisfactory results must ensure consistency of the mean Monte Carlo estimate of the PBLH with the original estimate, accompanied by appropriate uncertainty.

To achieve this, a **geostatistical modelling approach** will introduce elements of spatio-temporal autocorrelation and cross-correlation into the measurement equation, and subsequently integrate them into the synthetic profile generation procedure.

A **revised research question** can be expressed as follows: how to model the atmospheric vertical profile and measurement error to obtain satisfactory MC simulations for the computation of PBLH uncertainty?